

9. ELECTROSTATICS: The principle of fields superposition - answers

9.1. a) distance from Q_1 , between Q_1 and Q_2 : $x = r \frac{Q_1 - \sqrt{Q_1 Q_2}}{Q_1 - Q_2} = \frac{2}{3}r$

b) distance from Q_2 , outside Q_1 and Q_2 : $x = r \frac{Q_2 + \sqrt{Q_1 Q_2}}{Q_1 - Q_2} = r$

9.2. $q = \sqrt{\frac{mgx^3}{k\sqrt{4l^2-x^2}}}$

9.3. $\rho = \frac{\varepsilon \rho_0}{\varepsilon - 1}$

9.4. $q = \frac{4}{3}\pi\rho r^3 \sqrt{\frac{G}{k}}$

9.5. a) $E = \frac{8kq(d^2+4x^2)}{(d^2-4x^2)^2}; V = \frac{\pm 2kqx}{\frac{d^2}{4}-x^2}$

the sign of the potential is like the sign of the dipole charge closer to the test charge

b) $E = \frac{2kqxd}{(x^2-\frac{d^2}{4})^2}; V = \frac{\pm kqd}{x^2-\frac{d^2}{4}}$

c) $E = \frac{2kqd}{x^3} = \frac{2kM_e}{x^3}; V = \frac{\pm kM_e}{x^2}$, where $M_e = qd$ – dipole moment

In all cases: $F = Eq_1$

9.6. $E = \frac{kQd}{(x^2+\frac{d^2}{4})^{3/2}}$ and $V = 0$. When $x \gg d$: $E = \frac{kQd}{x^3} = \frac{2kM_e}{x^3}$

9.7. a) $E = 0; V = -3\sqrt{3}\frac{kq}{a};$ b) $E = 6\frac{kq}{a^2}; V = -\sqrt{3}\frac{kq}{a};$ c) $E = 6\frac{kq}{a^2}; V = \sqrt{3}\frac{kq}{a}$

9.8.* e.g. between charges q_2 and q_3 :

$$E = \frac{4k}{r^2} \sqrt{16(Q_3 - Q_2)^2 + \frac{1}{25}Q_1^2 + \frac{2}{5}Q_1(Q_3 - Q_2)\cos(\arctg 2)},$$

and in the middle of the hypotenuse $E = \frac{2k}{r^2} \sqrt{(Q_2 - Q_1)^2 + Q_3^2}$.

9.9. a) $E = 0;$ b) $E = 0;$ c) $E = 4\sqrt{2}\frac{kq}{a^2}$

9.10. $E = \frac{kq(a\sqrt{2}+2r)}{(a^2+ar\sqrt{2}+r^2)^{3/2}} - \frac{kq}{(r+a\sqrt{2})^2} - \frac{kq}{r^2}$

9.11.* $E = 0$

9.12. $E = \frac{2k\lambda}{R}; F = \frac{2k\lambda q}{R}$

9.13. a) $E = \frac{2\pi k\lambda h R}{(h^2+R^2)^{3/2}},$ b) $E = 0;$ maximum value of E for $h = \frac{\sqrt{2}}{2}R$

9.14. $E = \frac{kQ(x+\frac{1}{2}R)}{\left((x+\frac{1}{2}R)^2+R^2\right)^{3/2}} + \frac{kQ(x-\frac{1}{2}R)}{\left((x-\frac{1}{2}R)^2+R^2\right)^{3/2}}$

9.15. a) $E = \frac{\sigma}{2\varepsilon\varepsilon_0} \left(1 - \frac{h}{\sqrt{R^2+h^2}}\right);$ b) $E = \frac{\sigma}{2\varepsilon\varepsilon_0};$ c) $E = \frac{\sigma}{2\varepsilon\varepsilon_0}$

9.16. $E = \frac{\sigma h}{2\varepsilon\varepsilon_0} \left(\frac{1}{\sqrt{\frac{d^2}{4}+h^2}} - \frac{1}{\sqrt{\frac{D^2}{4}+h^2}} \right)$

9.17. $h = \sqrt{\frac{R^2 \left(1 - \frac{2mg\varepsilon_0}{\sigma q}\right)^2}{1 - \left(1 - \frac{2mg\varepsilon_0}{\sigma q}\right)^2}}$

9.20. $E = \frac{k\lambda}{x}$

9.18. $E = \frac{2k\lambda}{x}$

9.21. $F = \varepsilon\varepsilon_0 \frac{SU^2}{d^2}$

9.19.* $E = \frac{2k\lambda}{x} \cos \frac{\alpha+\beta}{2}$